A partial answer to this question is obtained on the basis of Eq. (6.1) whence it follows that, for $k_0 >> 1$, the effect of the $\tilde{\mathbf{V}}e^{2ik_0y}$ component of the first-approximation solution on sought solution $\mathbf{V}_{2c}$ is slight, so that two-dimensional idealization describes the required solution at least with an accuracy to $O(N^2)$. Actually, with an allowance for (4.1), the component of $(\mathbf{V}_1\cdot\nabla)\mathbf{V}_1$ from the right-hand side of (6.1) that is independent of y has the form

$$[(\mathbf{V}_1\cdot\nabla)\mathbf{V}_1]_c = \left(V_{cx}\frac{\partial V_{cx}}{\partial x} + V_{cz}\frac{\partial V_{cx}}{\partial z}\right)\mathbf{e}_x + \left(V_{cx}\frac{\partial V_{cz}}{\partial x} + V_{cz}\frac{\partial V_{cz}}{\partial z}\right)\mathbf{e}_z +$$
$$+\frac{1}{2}\left\{\left[\tilde{V}_x\left(\frac{\partial \tilde{V}_x}{\partial x} - 2k_0\tilde{V}_y\right) + \tilde{V}_z\frac{\partial \tilde{V}_x}{\partial z}\right]\mathbf{e}_x + \left[\tilde{V}_x\frac{\partial \tilde{V}_z}{\partial x} + \tilde{V}_z\left(\frac{\partial V_z}{\partial z} - 2k_0\tilde{V}_y\right)\right]\mathbf{e}_z\right\}.$$

Hence, with an allowance for estimates (5.2) and (5.3), it is evident that, although the $\tilde{\mathbf{V}}e^{2ik_0y}$ component of the first-approximation solution produces in principle Reynolds stresses, which affect the velocity field $\mathbf{V}_{2c}$, these stresses are nevertheless small for $k_0 >> 1$ in comparison with the term $(\mathbf{V}_c\cdot\nabla)\mathbf{V}_c$ (their ratio amounts to $\sim 1/k_0^2$), and they can be neglected.

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NONSTATIONARY VORTEX FLOWS OF AN IDEAL INCOMPRESSIBLE FLUID

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As is known, analytic methods of sufficiently general nature were developed only for potential motions in the two-dimensional hydrodynamics of an ideal incompressible fluid while vortex flows were investigated for quite particular cases [1, 2]. Examples of unbounded plane flows with concentration vorticity that allow analytic description of unbounded plane flows with concentration vorticity that allow analytic description are certain systems of point vortices, vortex pairs, Karman street [1], a three vortex system [3], as well as a Kirchhoff vortex which is an elliptical domain of homogeneous vorticity ω rotating at the angular velocity $\Omega = \omega AB/(A + B)^2$ (A, B are the ellipse semiaxes). Goerstner [1] obtained a unique exact solution for vortex flows with a free boundary which describes trochoidal waves on the surface of an infinitely deep fluid [1].

Such a type of plane nonstationary biharmonically time-dependent vortex motions of a fluid is found in this paper as includes elliptical vortices and Goerstner waves as particular cases and, exactly as potential flows, allows the method of conformal transformation for the solution of specific problems. It is shown that in a certain sense the class of motions found is exceptional, viz., out of all possible solutions in Lagrange variables that contain a finite set of time frequencies, only the two-frequency solution obtained in this paper satisfies the hydrodynamics equations. However, this class describes only such vortex flows for which a reference system can be indicated where the trajectories of the fluid particles remain localized, which is not satisfied, say, for the shear layer.

The theory developed for these flows is used to investigate the self-consistent interaction of a nonstationary vortex domain with an external potential flow.

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1. FUNDAMENTAL EQUATIONS

The equations of two-dimensional hydrodynamics of an ideal incompressible fluid in Lagrange variables are well known. These are the continuity equation

$$X_a Y_b - X_b Y_a = S_1(a, b) \tag{1.1}$$

and the equations of motion

$$X_{tt} X_a + Y_{tt} Y_a = -(1/\rho) p_a - \Phi_a, \quad X_{tt} X_b + Y_{tt} Y_b = -(1/\rho) p_b - \Phi_b, \tag{1.2}$$

where p is the pressure, ρ the density, Φ the secondary force potential, X, Y the coordinates of the fluid particle trajectory, a, b Cartesian Lagrange coordinates of a liquid element, t the time, and S_1 a certain function independent of the time; the subscripts denote differentiation with respect to the appropriate variable. These equations are rarely applied since their nonlinear terms enter in inconvenient form; however, they have the great advantage that their solution should be sought in a fixed domain of the variables a, b even in the presence of a free surface.

It is also known that the system of equations of motion is equivalent to a system of equations describing vortex conservation along a trajectory [4]

$$X_{tb} X_a - X_{ta} X_b + Y_{tb} Y_a - Y_{ta} Y_b = S_2(a, b) \tag{1.3}$$

[$S_2(a, b)$ is a function independent of the time]. This equation is substantially simpler than the originals since it does not contain the pressure and its order is not above the order of the system (1.2). In the interests of the investigation it is more convenient to write (1.1), (1.3) in complex form. We introduce the complex coordinate

$$W = X + iY \ (\overline{W} = X - iY)$$

and the complex argument

$$\eta = a + ib \ (\overline{\eta} = a - ib).$$

In these variables, (1.1) and (1.3) take the form

$$\frac{\partial}{\partial t}\left(W_\eta (\overline{W})_{\overline{\eta}} - W_{\overline{\eta}} (\overline{W})_\eta\right) = 0; \tag{1.4}$$

$$\frac{\partial}{\partial t}(W_{t\eta})(\overline{W})_{\overline{\eta}} - W_{t\overline{\eta}}(\overline{W})_\eta + (\overline{W})_{t\eta} W_{\overline{\eta}} - (\overline{W})_{t\overline{\eta}} W_\eta) = 0. \tag{1.5}$$

The equation of motion (1.5) allows certain simplification: By adding the continuity equation (1.5) differentiated with respect to the time, we obtain

$$\frac{\partial}{\partial t}\left(\overline{W}_{t\eta} (\overline{W})_{\overline{\eta}} - W_{t\overline{\eta}} (\overline{W})_\eta\right) = 0. \tag{1.6}$$

Thus, we have written the system of hydrodynamics equations in the form of two conservation laws for two Jacobians

$$D(W, \overline{W})/D(\eta, \overline{\eta}) = D(W_0, \overline{W}_0)/D(\eta, \overline{\eta}); \tag{1.7}$$

$$D(W_t, \overline{W})/D(\eta, \overline{\eta}) = D(W_{t0}, \overline{W}_0)/D(\eta, \overline{\eta}), \tag{1.8}$$

where W_0, W_{t0} are the complex coordinate and velocity at the initial instant. The compactness and graphic view of this mode of writing simplifies the investigation substantially

2. PTOLEMY FLOWS

In this section we obtain certain exact solutions of (1.4) and (1.6) and discuss their properties.

We shall seek the solution in the form

$$W = G(\eta) g(t) + F(\overline{\eta}) f(t). \tag{2.1}$$

There follows from the continuity equation

$$\frac{\partial}{\partial t}\left(|G_\eta|^2 |g|^2 - |F_{\overline{\eta}}|^2 |f|^2\right) = 0.$$

The expression in the parentheses is independent of the time in two cases: a) $|G_\eta|^2 = |F_{\bar\eta}|^2 = 1$, and the time functions satisfy the equality $|g|^2 - |f|^2 = \text{const} = C_1$, and therefore, one (g or f) is arbitrary, and b) $|g|^2 = |f|^2 = 1$ and the functions G and F are arbitrary.

Let us consider what constraints on the function is given by the equation of motion in both versions. Substituting (2.1) into (1.6), we obtain

$$|G_\eta|^2 g_t\bar{g} - |F_{\bar\eta}|^2 f_t\bar{f} = iC_2, \tag{2.2}$$

where C_2 is real. For the case a) we then have

$$g_t\bar{g} - f_t\bar{f} = iC_2,$$

from which we obtain the desired representation of the solution

$$W = \eta|g|e^{i\varphi(t)} + \bar\eta\sqrt{|g|^2 - C_1}e^{i\psi(t)}, \tag{2.3}$$

where $\varphi(t)$, $\psi(t)$ are the phases of the functions g and f connected by the relationship

$$\psi_t = (\varphi_t|g|^2 - C_2)/(|g|^2 - C_1). \tag{2.4}$$

The expression obtained for W describes a flow with constant vorticity. The form of the fluid particle trajectories is here determined by the law of variation g(t).

If it is assumed that a circle corresponds to the vortex motion domain in the plane of the Lagrange variables, then the solution (2.3), (2.4) describes an elliptical vortex that rotates with angular velocity $(\varphi + \psi)/2$ and is hence deformed so that its eccentricity ε is determined by the formula

$$\varepsilon = (|g| - \sqrt{|g|^2 - C_1})/(|g| + \sqrt{|g|^2 - C_1});$$

in the case $|g|$ = const the vortex is not deformed. Let us note that up to now only stationary domains with constant vorticity have been studied [5].

We now consider the second type of flow corresponding to the case b). It turns out to be substantially more distinct than the first, consequently, it is given special attention in this paper.

It follows from (2.2) that

$$W = G(\eta)e^{i\lambda t} + F(\bar\eta)e^{i\mu t} + h(t) \tag{2.5}$$

[λ, μ are real numbers] satisfies the system of hydrodynamic equations. We note that the functions G, F are arbitrary to a considerable extent (since the single constraint on their selection is the requirement that the Jacobian (1.7) not vanish), consequently, (2.5) describes a certain class of vortex flows. Let us study its properties.

We go over to a coordinate system that moves according to the law X = Re h(t), Y = Im h(t). Then the term h(t) vanishes in the expression for W. In such a reference system the particle trajectories are epicycloids (hypocycloids), i.e., the particles describe a circle whose center moves, in turn, along a circle. Hence, we would call this kind of flow Ptolemyan.

Two well known kinds of flows, Goerstner waves [1] and a stationary flow with constant vorticity in an elliptical domain [2] are particular cases of Ptolemy flows. Indeed, if $\lambda = 0$, $G(\eta) = \eta$, $F(\bar\eta) = -iR \exp(ik\bar\eta)$, $h(t) = 0$, are taken in (2.5), then we obtain expressions describing Goerstner waves:

$$X = a + Re^{kb}\sin(ka + \mu t),\quad Y = b - Re^{kb}\cos(ka + \mu t).$$

The case of the second flow (elliptical particle trajectories) is described by the formula

$$W = \alpha e^{ik\eta + i\lambda t} + \beta e^{-ik\bar\eta - i\lambda t},$$

where α, β, and k are constants.

We now consider certain singularities of Ptolemy flows. Uniform rotation of a fluid as a whole at an angular velocity Ω is characterized by the common factor $\exp(i\Omega t)$ in the expression for W. Consequently, by selecting an appropriate reference system, the time factors in the functions G and F can be altered. In such a procedure it is impossible to change just the difference in frequencies $\lambda - \mu$. In particular, the first term in the expression for W can be made time-independent.

Ptolemy flows are vortical. Their vorticity ω is written in Lagrange variables as follows:

$$\omega = 2\left(\lambda |G_\eta|^2 - \mu |F_{\bar\eta}|^2\right)/\left(|G_\eta|^2 - |F_{\bar\eta}|^2\right).$$

Naturally, it is time-independent. Let us find the velocity field corresponding to a Ptolemy flow. To do this it is necessary to eliminate η and $\bar\eta$ from the system

$$W = G(\eta)e^{i\lambda t} + F(\bar\eta)e^{i\mu t}, \quad V = i\lambda G(\eta)e^{i\lambda t} + i\mu F(\bar\eta)e^{i\mu t}$$

and to obtain an expression connecting the complex coordinate W to the complex velocity V. In sum, the velocity as a function of the coordinates is determined implicitly by the formula

$$F^{-1}\left(e^{-i\mu t}\frac{V - i\lambda W}{i(\mu - \lambda)}\right) = \overline{G^{-1}\left(e^{-i\lambda t}\frac{i\mu W - V}{i(\mu - \lambda)}\right)},$$

where F^{-1}, G^{-1} are the inverse functions of F and G, respectively.

It becomes evident that for all the simplicity of the time dependence of the particle trajectories, the flow field in Euler variables can turn out to be a quite complex function of the time.

Let us present the formula for the pressure in a domain with Ptolemy flow:

$$\frac{p}{\rho} = -\Phi + \frac{1}{2}\mu^2|F|^2 + \frac{1}{2}\lambda^2|G|^2 + \\ + \operatorname{Re}\left(e^{i(\lambda-\mu)t}\int\left(\lambda^2 G(\bar F)_\eta + \mu^2 G_\eta \bar F\right)d\eta\right).$$

Completing the survey of Ptolemy flow properties, we examine one singularity that makes this class of vortical nonstationary flows just as exceptional, in a certain sense, as are the potential flows. Namely, it includes all possible plane motions of a liquid with a finite number of harmonics in time (in the Lagrange variables). In other words, among the expressions for W described by a finite Fourier series

$$W = \sum_{k=1}^{N} Z_k(\eta, \bar\eta)\,e^{i\lambda_k t}$$

(λ_k are constants), only the biharmonic solutions (2.5) satisfy the hydrodynamics equations. Let us show this.

For N = 2 substituting the expression $W = Z_1 \exp(i\lambda_1 t) + Z_2 \exp(i\lambda_2 t)$ into the continuity equation results in the condition

$$D(Z_1, \bar Z_2)/D(\eta, \bar\eta) = 0,$$

which is satisfied if Z_1 is a function of $\bar Z_2$, or (equivalently) $Z_1 = G(\eta)$, $Z_2 = F(\bar\eta)$, as this is indeed written in (2.5).

For N = 3 we substitute

$$W = Z_1 \exp(i\lambda_1 t) + Z_2 \exp(i\lambda_2 t) + Z_3 \exp(i\lambda_3 t) \tag{2.6}$$

into the continuity equation and equate all the terms oscillating in time to zero. Two cases are possible.

A. All the frequency differences are not mutually equal. We then obtain

$$[Z_1, \bar Z_2] = 0, \ [Z_1, \bar Z_3] = 0, \ [Z_2, \bar Z_3] = 0,$$

where the square brackets denote the operation of taking the Jacobian in the variables η, $\bar\eta$.

There follows from the first condition that $\bar Z_2$ is a function of Z_1, and from the second that $\bar Z_3$ is a function of Z_1 but then we obtain from the last equality that $\bar Z_1$ is a function of Z_1. This is possible if and only if Z_1 is a complex function of one real parameter. Evidently Z_2 and Z_3 are also functions of just this same real parameter. We take this parameter as the Lagrange variable α. We have hence arrived at the assertion that W is a function of only α and is independent of b, and have thereby proved that a flow of the form (2.6) with nonequidistant spectrum cannot be two-dimensional. We obtained this result even without using the equations of motion.

B. The frequencies are equidistant. The result will be the same as in case "A." Now, however, it will be necesary to utilize both hydrodynamics equations. There follows from the continuity equation

$$[Z_1, \overline{Z}_3] = 0, \ [Z_1, \overline{Z}_2] + [Z_2, \overline{Z}_3] = 0,$$

and from the equations of motion

$$\lambda_1 [Z_1, \overline{Z}_2] + \lambda_2 [Z_2, \overline{Z}_3] = 0,$$

and again, as in case "A"

$$[Z_1, \overline{Z}_2] = 0, \ [Z_1, \overline{Z}_3] = 0, \ [Z_2, \overline{Z}_3] = 0, \quad \text{if} \quad \lambda_1 \neq \lambda_2.$$

Therefore, W is again a function of just a and is independent of b.

It can be shown in an analogous manner that flows with trajectories describable by N frequency functions of the time cannot be two-dimensional, and therefore, the Ptolemy flows (2.5) are exceptions to this rule.

3. MERGER OF PTOLEMY AND POTENTIAL FLOWS

The problem of merging the vortical and potential flow domains has its own, at first glance, insuperable difficulties. In fact, the problem of the potential flow around a given boundary of arbitrary shape is not solved in the general case. The problem of the potential flow around an arbitrary domain with a Ptolemy flow is still more complicated since the reverse influence of the potential flow on the shape of the boundary and on the motion of the domain with vorticity as a whole should be taken into account. Nevertheless we succeeded in solving it for the case of a simply connected domain with a Ptolemy flow.

The problem is formulated as follows. The Ptolemy flow (2.5) is given in the domain $b \geq 0$. It is required to determine the potential flow outside this domain. We require compliance with the continuity condition by the total velocity on the merge boundary.

The potential motion in the exterior of the vortical domain is then represented in parametric form. Indeed, the relationships

$$\begin{aligned} W &= G(\eta) e^{i\lambda t} + F(\eta) e^{i\mu t} + h(t), \\ V &= i\lambda G(\overline{\eta}) e^{i\lambda t} + i\mu F(\overline{\eta}) e^{i\mu t} + h_t(t) \end{aligned} \tag{3.1}$$

($\eta = a + ib$, $b \leq 0$) solve the problem. These expressions are a parametric mode of writing the potential flow since it follows from them that $\overline{V} = \overline{V}(W, t)$, while the continuity conditions for the shift in the vortex and pressure boundaries are satisfied for b = 0 (continuity of the pressure is verified by direct evaluation).*

It should be noted that the proposed mode of writing the potential flow does not agree with either the Lagrange or the Euler modes.

The transition from a parametric to an Euler representation is evident and accomplished by eliminating η from the system (3.1). In order to go over to the Lagrangian description, we consider the parameter η time-dependent, and its derivative with respect to time is determined by the expression

$$\eta_t = (V(\overline{\eta}, t) - W_t(\eta, t)) / W_{\eta}(\eta, t),$$

which forms, together with its complex-conjugate, a system of two equations in η and $\overline{\eta}$. By integrating it a dependence of the quantity η on the time and the initial conditions η_0 can be found which are selected as Lagrange coordinates.

We now apply the results of investigating the general properties of Ptolemy flows to study specific vortex streams.

$$W = G\left(e^{ik\eta}\right) e^{i\lambda t} + F\left(e^{-ik\overline{\eta}}\right) e^{i\mu t} + h(t)$$

*Let us note that the absence of branch points in the function W, i.e., its derivative does not vanish, is the uniqueness condition for the potential flow velocity field.

4. SELF-CONSISTENT INTERACTION BETWEEN A NONSTATIONARY VORTEX DOMAIN AND THE SURROUNDING POTENTIAL FLOW

Let us consider a single vortex domain around which the flow is potential externally. We assume that the expression

$$W = G(e^{ik\eta})e^{i\lambda t} + F(e^{-ik\bar{\eta}})e^{i\mu t} + h(t)$$

gives the Ptolemyan vortex motion within this domain. The interior of a unit circle corresponds to it on the plane of the Lagrange variable exp (ikη), which is equivalent to the condition $b \geq 0$. We consider the functions G and F analytic and without singularities. On the flow merger boundary $b = 0$, consequently, in conformity with (3.1) the potential flow in the exterior of the domain is written in the form

$$\begin{aligned} W &= G(e^{ik\eta})e^{i\lambda t} + F(e^{-ik\eta})e^{i\mu t} + h(t), \\ V &= i\lambda G(e^{ik\bar{\eta}})e^{i\lambda t} + i\mu F(e^{-ik\bar{\eta}})e^{i\mu t} + h_t(t), \end{aligned} \tag{4.1}$$

where $\eta = a + ib$, $b \leq 0$. At infinity, the fluid is at rest, i.e., $V \to 0$ as $W \to \infty$. Consequently, we should set $\mu = 0$, $h_t(t) = 0$ in the relationships (4.1) [for simplicity we take $h(t) = 0$].

We have thus obtained a solution describing the potential flow around the vortex domain under consideration: The vortical flow is described by the expression

$$W = G(e^{ik\eta})e^{i\lambda t} + F(e^{-ik\bar{\eta}}), \tag{4.2}$$

while the potential flow is given parametrically

$$\begin{aligned} W &= G(e^{ik\eta})e^{i\lambda t} + F(e^{-ik\eta}), \\ V &= i\lambda G(e^{ik\bar{\eta}})e^{i\lambda t}. \end{aligned} \tag{4.3}$$

It is seen that the shape of the vortex domain is determined just by the interrelation between the functions F and G, consequently, the function $G(\eta)$ can be selected from considerations of convenience, for instance, in the form $G(\eta) = \alpha \exp(ik\eta)$.

The known exact solution for a Kirchhoff elliptical vortex is obtained from (4.2) and (4.3) if we take

$$G(\eta) = (1/2)(A + B)\exp(ik\eta),\quad F(\bar{\eta}) = (1/2)(A - B)\exp(-ik\bar{\eta}),$$

where A, B are the ellipse semi-axes.

We also indicate that the relationships (4.2) and (4.3) yield a solution of the initial problem when the velocities, governed uniqely by the shape of the contour are given on a contour of arbitrary shape to a considerable degree.

It is more customary to give the initial shape of the vortex contour W_0^* and the velocity on it W_{t0}^* in Euler coordinates. In this case, the determination of the functions G and F reduces to seeking the conformal mapping χ of the unit circle on the exterior of the domain W_0^*. Then the vortex boundary is written in the form

$$W_0^*(e^{ika}) = \chi(e^{ika}) = \chi_1 e^{ika} + \sum_{n=0}^{\infty} \chi_n e^{-ikna},$$

where χ_n are Fourier series coefficients, and n is an integer.

Let us mention that the parametric representation we selected for the boundary is unique, which assures a velocity drop at long ranges from the vortex that is inversely proportional to the radius.

The flow within the vortex domain is determined by the expression

$$W = \chi_1 e^{ik\eta + i\lambda t} + \sum_{n=0}^{\infty} \chi_n e^{-ikn\bar{\eta}}, \tag{4.4}$$

which is valid for a given W_{t0}^* if

$$W_{t0}^* = i\lambda\chi_1 e^{ika + i\lambda t}.$$

The value of λ is found from the relationship

$$\lambda = -\frac{1}{2\pi}|\chi_1|^{-2}\,\mathrm{Re}\oint_{W_0^*} \overline{W}_{t0}^* dW.$$

The functions G and F defined by (4.4) should satisfy the condition that the Jacobian (1.7) does not vanish. Consequently, the problem with a given vortex domain shape at the initial time is not always solvable by using (4.2)-(4.4).

Let us note that the vortex shape and the vorticity within it (but again, not arbitrarily) can be given as initial data.

Let us consider specific examples. For simplicity, we seek a class of stationary vortices that are not deformed but just rotate (there is no translational motion). The fluid particle trajectories forming the boundary of the vortex domain b = 0 in a reference system rotating at the vortex angular velocity Ω will coincide with the vortex boundary and their velocity is tangent to it so that

$$W_t = \gamma W_a. \tag{4.5}$$

Here γ is a real constant (in the general case γ depends on a but a W* can always be selected such that $\gamma(a)W_a = \gamma^* W_a^*$, where γ^* is independent of a).

In the rotating coordinate system the solution (4.2) becomes

$$W = \alpha\, e^{ik\eta + i(\lambda-\Omega)t} + F\left(e^{-ik\bar{\eta}}\right) e^{-i\Omega t}.$$

Substituting this expression into condition (4.5), we obtain that F is a power-law function and (4.3) can be written in the form

$$W = \alpha\, e^{ik\eta + i\lambda t} + \beta\, e^{-ikl\bar{\eta}}, \tag{4.6}$$

where l is a non-negative integer. It describes the family of stationary vortices. For l = 1 we obtain an elliptical vortex; vortices corresponding to the values $l \geq 2$ represent domains of hypocycloidal shape with a number $l + 1$ of cusps, and rotate as a whole with an angular velocity $\Omega = l\lambda(l + 1)$. The condition of no self-intersections of the vortex boundary is the inequality $\beta \leq \alpha l^{-1}$. This condition assures the univalency of the potential flow velocity field.

We write the flow vorticity (4.6) as

$$\omega = \frac{2\lambda\alpha^2}{\alpha^2 - l^2\beta^2 \exp\left[-2k(l-1)b\right]}.$$

If the vorticity is homogeneous for the elliptical vortex, then it is minimal at the vortex center and grows towards the boundary for the remaining terms of the family (4.6).

In conclusion, let us mention that the vortex domain for a function F different from a power law is deformed in a sufficiently complex manner in addition to rotation. The nature of this deformation is easily determined from (4.2).

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